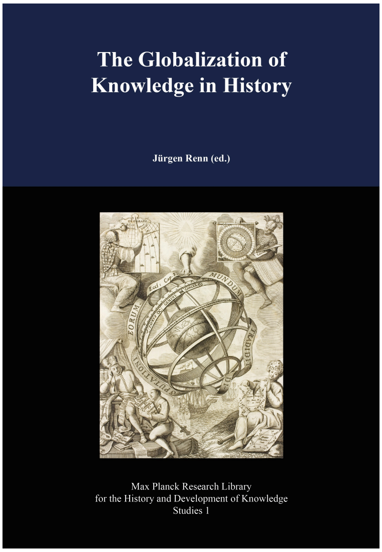
Max Planck Research Library for the History and Development of Knowledge

Studies 1

Peter Damerow:

The Origins of Writing and Arithmetic



In: Jürgen Renn (ed.): *The Globalization of Knowledge in History*
Online version at http://edition-open-access.de/studies/1/

ISBN 9783844222388
First published 2012 by Edition Open Access, Max Planck Institute for the History of Science under

Printed and distributed by:
Neopubli GmbH, Berlin
http://www.epubli.de/shop/buch/17018

The Deutsche Nationalbibliothek lists this publication in the Deutsche Nationalbibliografie; detailed bibliographic data are available in the Internet at http://dnb.d-nb.de

Chapter 6
The Origins of Writing and Arithmetic

Peter Damerow

6.1 Globalized Systems of Writing and Arithmetic

Writing and arithmetic are cultural techniques which are essential conditions of the organization of modern societies. They are usually considered as distinct human activities with distinct origins. However, recent work based on archaeological evidence suggests a common origin.[1] It is a basic claim of the present paper that there were close relations between writing and arithmetic at the time of their emergence.

Both writing and arithmetic are based on operations with systems of symbols that represent *cognitive constructions*, either directly or indirectly.[2] The main reason for considering writing and arithmetic to be relatively independent of each other is that the cognitive constructions they represent and the way in which they represent them are different.

Developed writing systems are predominantly glottographic. They represent language by some kind of phonetic coding.[3] Such writing systems are based on a

---

[1]The analysis of the origins of writing and arithmetic presented here focuses on the development and application of the cuneiform writing system in the ancient Near East. Due to the durability of clay tablets used as writing material, the excavated tablets and other artifacts from this region provide an abundance of information, revealing the development from the precursors of writing and arithmetic in the fourth millennium BCE to the spreading of the technology of writing throughout the Mediterranean area,see (Sasson 1995, vol. 4), and the creation of Babylonian mathematics, see (Robson 2008).

[2]The term *cognitive construction* is used here in the widest sense, and includes all forms of the mental organization of feelings, perceptions, beliefs and thoughts. Such mental constructions are usually organized in cognitive structures consisting of objects represented by mental images or conceptual structures together with mental operations related to them, systems which may be called *mental models*, see (Renn and Damerow 2007) and also the introduction to this volume (chapter 1). Mental constructions and models can be externally represented by symbols and symbol systems. The relation between mental constructions and external symbols may be called *iconic* if the symbol somehow depicts a mental image, or *conventional* if the symbol represents a mental construction only by arbitrary definition. Writing and arithmetic are both tools for the external symbolic representation of mental constructions and models. They both use primarily conventionally defined symbol systems.

[3]We follow Hyman (2006), who offers a conceptual clarification of the various types of relations between written texts and their meaning, with specific focus on the development of early writing systems. Glottographic representation of meaning is based on the phonetic coding of language. It is closely linked to reading since the representation determines the chain of utterances of words.

Figure 6.1: Obverse, edge and reverse of an archaic Babylonian bookkeeping record (ca. 3200-3000 BCE).

standardized and stable inventory of graphemes representing words, morphemes, syllables or phonemes. This inventory has to be historically transmitted by a social group and to be rich enough to represent an essential part of the language spoken by this group. According to this definition, cognitive constructions are represented by writing only indirectly, insofar as they are expressed in language.

By contrast, arithmetical systems are non-glottographic, representing cognitive constructions such as numbers and numerical operations directly.[4] They are based on a standardized and stable inventory including graphemes designating quantities and rules for performing arithmetical operations. This inventory has to be historically transmitted by a social group and be rich enough to perform basic calculations comprising additions and multiplications.[5]

By contrast, a non-glottographic representation of meaning is closely related to verbalizing. In this case, symbols or chains of symbols determine iconically or conventionally their meaning without determining how this meaning has to be verbalized.

[4]For a systematic, theoretical reconstruction of the development of knowledge representation structures reflecting arithmetical operations, see (Damerow 2007).

[5]The non-glottographic representation of cognitive constructions constituting arithmetic does not exclude any significant role language may play for building these up and transmitting them between members of a social group. Language may serve as a means to make the rules of numerical operations explicit. Language may also serve as a means to conceptualize and communicate the cognitive context of such rules, in particular, systems of numbers. As far as the relation

Numerous systems of writing and of arithmetic developed in history and spread geographically until they reached the level of globalization characteristic of modern societies.[6] Given the great variety of different systems of writing and arithmetic, this globalization process was anything but a simple process of diffusion. Throughout the world, local developments of writing and arithmetic have interacted with each other in various ways. In the case of arithmetic, the final outcome is a relatively unified system of arithmetical notation and calculation methods. In the case of writing, the situation is different. Today, as a result of globalization processes, writing is used all over the world, but neither the languages nor the writing systems have been unified by these processes.[7]

The globalization processes of writing and arithmetic, which resulted in the present situation, are far from being adequately investigated or well understood. Neither in the case of writing nor of arithmetic is it clear to what extent globalization is the result of transfer and diffusion of knowledge from one place of origin to other regions of the world, and to what extent it is the result of independent developments that interacted with each other and merged into current systems of writing and arithmetic.

6.2 When is Writing Writing and When Is Arithmetic Arithmetic?

To investigate the early phases of the emergence, development, transfer, diffusion and, finally, globalization of writing and arithmetic, the different types of knowledge that evolved over the course of this process need to be identified. Writing and arithmetic have been characterized above as knowledge representation structures that are shared and historically transmitted by certain social groups or populations. They are external representations of mental constructions that over the course of their development, as will become clear in the following, became increasingly different and independent of each other. What they do have in common is that the media of these representations are constituted by conventionally defined symbol systems.

---

of symbolic representation and language is concerned, arithmetic is thus in a certain sense the opposite of writing. Writing refers to cognitive constructions by operations within a symbolic system that represents language by phonetic coding. Arithmetic refers to cognitive constructions by operations within a symbolic system, which represent these cognitive constructions themselves by arithmetical symbols and symbolic operations, while language is used only to conceptualize and verbalize these operations.

[6] Writing systems and arithmetical systems both exist in different forms using different means of symbolic representation. Different languages may be represented by different writing systems, or by the same writing system with the same or with partly different phonetic coding. Correspondingly, different arithmetical systems may be represented by the same or by different systems of symbolic notation. Even the same arithmetical system may be represented quite differently in different contexts.

[7] A further step in the process of globalization, however, may create in a multilingual context a so-called “lingua franca.” In certain areas of application, such as the field of modern sciences, the English language in combination with Latin characters developed into a kind of written lingua franca, facilitating the scientific documentation and communication of scientific knowledge.

Of course, this common basis of writing and arithmetic is not specific to such symbol systems. Their use, transmitted by rituals or instruction, goes back some 10,000 to 40,000 years to the Upper Paleolithic Revolution, that is, to a prehistoric period of mankind, long before the occurrence of a technology that can be interpreted as writing or arithmetic as we know it.[8] In order to understand how writing and arithmetic emerged, we must study the specific kinds of symbolic representation that actually contributed to their simultaneous emergence in the Early Bronze Age.[9]

Which specific characteristics of those kinds of symbolic representation demarcate the onset of the development of writing and arithmetic? Any investigation of the early development of writing and arithmetic faces a problem: the cognitive constructions they represent were constituted by historically changing sets of mental operations with varying areas of application. Even if a certain symbolic representation depicts the same object or setting over a long time, its meaning may have altered substantially according to the changing cognitive constructions that determine its symbolic meaning, on the one hand, and to which the object or setting is mentally assimilated, on the other. The investigation of the origins of writing and arithmetic therefore requires some conceptual clarification of the specific kinds of cognitive constructions that formed the basis for the emergence of writing and arithmetic.

Writing: From a modern point of view, it makes sense to define writing as a glottographic representation of spoken language by phonetic coding in a lasting medium. This definition makes sense in a globalized context in which writing is essentially a universally applied means to represent and communicate all forms of knowledge. Given that this function of writing is in fact the major characteristic of its modern use, it is an abstraction from its numerous other functions,[10] in particular from its various functions in different ranges of application, and in different cognitive contexts. However, at the early stages of its development, writing co-developed with certain areas of the social reality of the time, such as economical redistribution, multilingualism, foreign trade, religious rituals or the training of officials. It is precisely the development of the relation between the changing ranges of application to real objects and the changing knowledge about them that accounts for the great variety of the early developing writing systems. The modern definition is indifferent to such conditions. Thus, it comes as no surprise that the kinds of geographically and historically changing representations of language in the early phases of the development of writing that may be considered as writing in the modern sense are a controversial issue.[11]

---

[8] For a detailed study of the early use of symbols, see (Leroi-Gourhan 1993).

[9] See (Damerow 1998) for a reconstruction of the cognitive processes involved in the prehistoric development of symbolic representation.

[10] For the variety of different functions of writing, see chapter 5.

[11] In the sequel to the influential study of Gelb (1952), several attempts have been made to classify the various kinds of symbolic representation involved in the early development of writing, and that left traces distinguishing writing systems based on Latin characters from other writing systems

Arithmetic: The situation is similar in the case of arithmetic. From a modern point of view, arithmetic can be defined as a non-glottographic representation of numbers and numerical operations by symbols and symbol transformation rules. Again, this definition makes sense in a globalized context in which arithmetic is essentially a universally applied means of representing and communicating the handling of counted or measured quantities of arbitrary objects. The definition refers to an abstract concept of number. It does not distinguish between different ranges of application and different cognitive contexts. At the early stages of its development, however, arithmetic also co-developed with certain areas of the social reality of the time, such as the accumulation, trading and redistribution of commodities in stratified societies, the training of administrators or the institutionalization of early scholarship, such as Babylonian astronomy, Platonic philosophy or Euclidean mathematics. It is the development of techniques for handling quantities and for reflecting on and symbolizing the operations with them that accounts for the great variety of the early developing systems of numerical operations, some of which survived among recent indigenous cultures comparable to those of the Stone Age. Before they had contact with the modern world, the tribes of Australia and South America did not count beyond three. Other indigenous cultures used extended counting techniques, such as the body counting of the natives of New Guinea, and sometimes also used tallies to control quantities. But the areas in which these techniques were applied were narrowly restricted. Moreover, these techniques did not necessarily include numerical operations, such as additions and multiplications, which today are associated with any number concept.[12] Since the modern definition of arithmetic is heavily influenced by the Platonic and Kantian tradition, according to which the number concept is an a priori concept, not resulting from experience, it is not affected by the historically and geographically changing cognitive constructions on which numerical operations were based. Thus, this definition does not enable a decision about which early arithmetical techniques indicate knowledge about numbers in the modern sense.

In view of these ambiguities concerning the common concepts of writing and arithmetic, we will distinguish here systematically between proto-writing and writing, and also between proto-arithmetic and arithmetic. The term *proto-writing* will

---

such as Chinese, Hebrew or Arabic; see, for example, the second chapter of (DeFrancis 1989, 20–64). From his taxonomy, Gelb speculatively derived a universal sequence of the development from non-glottographic symbols to glottographic writing, which led him to erroneous claims such as that the then still undeciphered writing system of the Maya could not be based on phonetic coding (Gelb 1952, 54–59), a claim that, after the successful decipherment of the Mayan writing system, turned out to be fallacious. While the classification of writing systems and their constituents provides helpful tools for the description of the differences between them, such a classification in itself contributes little to the understanding of the historical processes that determined their emergence, development and globalization. For a thorough critique of the interpretation of typological ideals as an evolutionary stage, see (Michalowski 1994).

[12]For a theoretical reconstruction of the cognitive background of such techniques, see the classical study of Wertheimer (1925) and the extended analysis of Damerow (1996, in particular chapter 9).

be used to designate the *non-glottographic* symbol systems that historically precede writing and share some functions with the writing system they precede, but that could not have been used to represent, independent of context, the flow of free speech.[13] The term *proto-arithmetic* will be used to designate symbol systems, such as tallies or units of counting, for controlling quantities in direct relationship with the objects and symbols that represent these objects. Such systems of symbols do not represent any kind of context-independent numbers reflecting accumulated experiences achieved in the process of controlling quantities by means of correspondences. Their invention and use may have resulted in the development of arithmetical techniques, but they can be used for controlling quantities of objects, even without any cognitive numerical construct.

6.3 The Emergence of Proto-Cuneiform Bookkeeping in the Ancient Near East

The following analysis of the origins of writing and arithmetic will focus on the development of cuneiform writing in Mesopotamia. There are two reasons to analyze specifically the early development of the cuneiform writing system.

First, cuneiform writing, as far as we know, is the earliest writing system in the world.[14] During the roughly 3,000 years of its use, it spread to many regions of the Near East and moreover influenced directly or indirectly the development of numerous other writing systems used in the Mediterranean area and the Western part of the Eurasian continent.

Second, due to the durability of clay as a writing medium, the early history of the cuneiform writing system and its possible precursors are documented by an abundance of archaeological findings. Moreover, a long tradition of archaeological and philological research contributes to the existing knowledge in this field of scholarly study, so that the answers to many questions are not dependent on more or less risky speculations on historical opportunities and possibilities.

Two major kinds of symbolic representation used in Mesopotamia and in neighboring regions have been considered as immediate precursors of writing. One is the use of *seal impressions* on the stoppers of storage jars, door locks and other means of securing property. They indicate ownership by symbolically representing the owner, or ensure some kind of legal binding by symbolically representing the person or institution that controls, through influence or power, the adherence to the social behavior signified by the seal impression. Such seal impressions were produced by stamp and cylinder seals. They were invented in the fourth millennium BCE in Mesopotamia, and later adopted in Egypt and by the Indus civilization.

[13] It should be noted that this definition does not exclude that phonetic coding was used for specific purposes.

[14] This is obviously true for the early writing systems of China and Meso-America, which were created independently, but developed or at least attested only much later. The situation is less clear in the case of Egyptian hieroglyphs. See the discussion of the earliest attestations of Egyptian writing by John Baines in (Houston 2004).

The representation is partly iconic, depicting persons, objects, mythological figures or complex scenes, but it seems obvious that their main reference to the social setting in which they were used was conventionally determined by the activities and transactions with regard to which they functioned.[15]

The other kind of symbolic representation that contributed directly to the invention of writing is a certain use of small, geometrically shaped tokens made from clay. Thousands of such tokens have been found at archaeological sites scattered over the regions of the Near East. The function of these tokens remained obscure until their connection to the origins of cuneiform writing was discovered. This connection is still the subject of controversial debate.[16] There is, however, a basic agreement that at least in the second half of the fourth millennium BCE, such tokens were used as counters, that is, they were used in direct relationship with objects or units of measurement.

Some of the tokens look like icons of the objects they may have represented. They are shaped like small models of these objects (animals, containers, and so forth). The shapes of some of them resemble signs of the later script, suggesting that they had a function similar to that of logograms of early writing systems. Most of the tokens, however, are completely abstract (spheres, cylinders, cones, tetrahedrons, lenses, discs, pellets). Their relation to the objects they may have represented must have been determined merely by conventions concerning their use in certain contexts. In the second half of the fourth millennium BCE, combinations of equal or partially different tokens were occasionally included in closed and sealed hollow clay balls, securing as bullae the information represented by these combinations. While such closed assemblages would obviously represent significant indicators of the ultimate arithmetic meaning of early clay markers, the evidence from opened or scanned clay balls is so meager as to be discountable. Thus, tokens shaped like models of objects have not been demonstrated to have been included among such assemblages; nor can we state with any confidence whether counts of simple tokens within clay balls exceeded some number representing bundling units in the proto-cuneiform records (generally either six or ten, dependent on the numerical system involved); and finally, combinations of differently shaped tokens in the balls do not show regularities that would indicate the representation of standardized numerical systems.[17]

Nevertheless, there can be no doubt that the tokens were actually used as counters. The strongest indication for this use is provided by marks that were sometimes impressed into the moist clay surfaces of the bullae using a stylus, fingers, or the tokens themselves. With few exceptions, such impressions correspond precisely to the tokens inside; they map combinations of impressions to combi-

[15] See the interesting attempt to reconstruct the hierarchy of the administration of Susa before the invention of writing from the application of sealings and their motifs by Dittmann (1986).

[16] For an extensive documentation of such tokens, see (Schmandt-Besserat 1992). Her classifications of these tokens, the attribution to specific archaeological layers and thus her datings, as well as her speculative interpretation of their functions, however, have met with severe criticism.

[17] See (Bauer et al. 1998, 46–56; Englund 2006; Damerow and Englund forthcoming).

nations of counters. Furthermore, the arrangements of such impressions resemble numerical notations in the later script. Summing up these findings, the tokens were used as counters, but cuneiform lacks the essential attributes of abstract numbers. They were used as proto-arithmetical tools in the sense defined above.

The archaeological findings show another innovation which occurred around the same time as the sealed bullae: small sealed clay tablets bearing combinations of marks similar to those sometimes impressed into the surface of bullae. These so-called *numerical tablets* share with the counters the lack of indications for standardized numerical systems. These so-called numerical tablets seem to share with the counters an ambivalence to the standardization of numerical systems. For instance, numerical signs were repeated more than nine times in some documents from Jebel Aruda. This indicates that in those documents the signs may still have represented the real objects or containers, although these sign clusters were themselves embedded in strings of numerical signs, suggesting the full notations reflected an advanced system of numerical bundling (Bauer et al. 1998, 50–51 and 214).

Around the end of the fourth millennium BCE, another innovation was introduced which was key to the development of writing and arithmetic. Clay tablets found in Uruk in southern Mesopotamia, in Susa in the region of Khūzestān, and (one example) in Godin Tepe in the Zagros mountains of Iran, as well as a seal impression and a numerical notation, display one or two graphemes drawn with a stylus onto the moist clay. These graphemes on *numero-ideographic tablets* indicated the object, the quantity of which was registered by the numerical notation.

The invention of graphemes complementing seal impressions and numerical notations offered virtually unlimited opportunities for representing structured information. It was much easier to invent a new grapheme than to carve a new seal. Furthermore, by using graphemes and dividing the tablets into different fields, more information could be placed on one tablet than was previously possible. These opportunities were soon used extensively. Hundreds of different graphemes were invented. These were standardized, at least partly, to represent objects, persons, institutions, types of transactions, and so forth. The tablets were divided by lines into hierarchically ordered fields, each one containing a specific entry providing information about some economic activity. The nearly 5,000 extant tablets and fragments of such *proto-cuneiform adminstrative texts* represent the earliest form and, at the same time, a transient stage of the development of both cuneiform writing and Babylonian arithmetic.

6.4 The Inherited Semantics of Proto-Cuneiform Administrative Tablets

Proto-cuneiform writing inherited from its preliterate precursors its area of application as an administrative tool and its functions within the context of administrative control. The preliterate administrative tools (seal impressions, counters, stylus impressions representing counters) were used to control activities such as various kinds of transfer of economical resources and products. In order to control these activities, information had to be available about the four types of conditions that determined an activity of this kind: the kind of resource or product involved; the amount of this resource or product; the agent concerned by the activity; and the official or office in the administration responsible for controlling the activity. Precisely these four conditions are the main semantic categories of proto-cuneiform administrative tablets.

The administrators using the preliterate precursors or these tablets (numerical tablets with seal impressions and numerical notations) to record such conditions were unable to document them in a way that the resulting documents could be interpreted independently of the context. In order to interpret, for instance, a numerical tablet or sealed bullae with counters, and to derive the specific information about who authorized what with these documents, a number of conditions have to be known: the owner of the seal; the kind of activity he was responsible for; the type of product related to the specific activity; the procedure of counting or measuring the amount of the product applied in this case; and the function of the document.

The numero-ideographic tablets made one of these implicit categories of information explicit by introducing graphemes for the objects of the documented activities, thus paving the way for the invention of proto-cuneiform. In the developed proto-cuneiform system, this inherited category was the general semantic class of many graphemes with iconic relations to various resources and products.

This did not lead, however, to a logographic archetype of cuneiform writing, as was often taken for granted (Gelb 1952). The use of graphemes was not generalized from words or morphemes of the Sumerian (or any other) language, but rather from implicit and explicit semantic categories of bookkeeping practices; graphemes did not represent these categories independent of such practices. Proto-cuneiform was developed to improve the functions of its preliterate precursors. This required a greater variety of semantic coding than the simple matching of graphemes with objects. Thus, agents, officials and offices, in particular, were no longer represented predominantly by seal impressions and the context of their use, but by newly created sign combinations.

A statistical analysis has shown that the number of such sign combinations is much higher than the number of uses that could be interpreted as logographic (Damerow and Englund forthcoming). The subject of this analysis was a sample of eighty-six closely related tablets and tablet fragments: the tablets of the former Erlenmeyer collection.[18] The sample tablets contain about 780 entries. The number of different signs and sign combinations of these entries representing products

[18]The collection, preliminarily published in (Nissen et al. 1993) and electronically accessible at the CDLI website, http://cdli.ucla.edu (search for the primary publication *MSVO 3*), turned out to be highly significant for our understanding of the sign combinations representing agents of economical transactions. In 1989 they were auctioned off; the auction at Christie's in London included lots with several artifacts each. The majority of the tablets were purchased by the

is less than thirty, but the number of different sign combinations representing agents concerned with the registered activities is greater than 300.

The significance of this result in revealing the extent to which proto-cuneiform writing represents language patterns becomes evident if we extrapolate the figures to the total corpus of the more than 6,600 proto-cuneiform tablets and tablet fragments. This corpus contains close to 40,000 entries.[19] Assuming that the statistical relations are roughly similar to those in the analyzed group of sample texts, we have to expect more than 23,000 different sign combinations representing agents. If proto-cuneiform sign combinations should, in fact, represent language patterns, these sign combinations representing agents are evidently the candidates for phonetic coding. However, in spite of the fact that in many cases the phonetic values of the corresponding signs of later cuneiform writing are known, the attempts to interpret the sign combinations as phonetically coded Sumerian or Akkadian names, or designations of institutions, failed. After some eighty years of work on the question of the language affiliation of the proto-cuneiform corpus, the debates surrounding it still focus on less than ten examples of alleged phonetic readings.

The emergence of proto-cuneiform brought about innovative new technologies, also with regard to how quantities of resources and products were controlled. The numerical signs of proto-cuneiform tablets are now highly standardized and organized in numerical systems with standardized relations between the different units. Combinations of units were converted into a standardized form by replacing repeated numerical signs by signs with a higher value as soon as the value they represent was reached.

The resulting numerical notations, which often represent hundreds of thousands of units, seem to correspond perfectly to the later tradition of the arithmetic of cuneiform writing. This impression, however, is misleading. The standardized numerical systems inherited the lack of differentiation of quantity and quality from the context-dependent use of their precursors. This resulted in a short-lived transitional system of proto-arithmetic, which is unparalleled by any other numerical system in the world. The basic numerical signs of the proto-cuneiform administrative documents changed their numerical values depending on the quantified objects, or more precisely, on the units of the metrological system of their quantification. The fact that the values of the signs changed so radically that not even the order of their sizes was kept constant considerably reduced the ambiguity resulting from the context-dependency of the numerical notations.

A further characteristic of the numerical notations on the proto-cuneiform tablets evidencing their context-dependency is the smooth transition between numerical and non-numerical proto-cuneiform signs. Numerical signs were partly

Land Berlin and are now on permanent loan to the Vorderasiatisches Museum; the rest of the collection is distributed among three other museums and some private collectors.

[19] See the contribution by Englund in (Bauer et al. 1998, 65–81).

used also to designate objects, and the incised pictographs of cuneiform writing, if used in the numerical context of an account, could represent numerical values.

According to the dependency of numerical values on individual contexts, there existed no context-independent techniques of performing calculations. *Additions* could be performed with the numerical impressions as they were performed with counters. *Multiplication* did not exist as a generally applicable calculation technique. However, three types of operations can be identified in proto-cuneiform administrative documents, which can be considered as precursors of the multiplication technique.

The *first* type is to reproduce a quantity several times, for instance, to get from an amount of grain for one day the amount for a month of thirty days. From an arithmetical point of view, this operation corresponds to a multiplication with a small natural number. Such an operation could easily be performed by repeated addition.

The *second* type of multiplication depends on a numerical relation between two quantities, such as the rule that for the production of three pieces of a certain grain product, five units of barley were required. By applying the first type of operation equally to both quantities, further values with the same relation could be achieved. From an arithmetical point of view, such calculations correspond to multiplications with a fraction; in the given example the multiplication with five over three, but these factors remained implicit and were never written.

The *third* type of operation corresponding to the later multiplication technique was the most sophisticated one. Accordingly, the results documented by administrative documents contain a remarkably high percentage of errors. This type of operation was used exclusively by surveyors to calculate the areas of fields from measurements of the lengths of their sides, applying to irregular quadrangles what is known from much later history as the surveyor's formula. Arithmetically, this operation corresponds to multiplying the means of opposite sides. Since the system of length measurements and the system of area measurements were not coordinated with each other, the procedure had to be specific for this single purpose and could not be applied to any other type of problem.

Thus, the heritage of preliterate administrative tools determined not only the area of application and the functions of the proto-cuneiform writing system, but moreover the detailed semantics of its sign combinations. The outcome was a historically unique, integrated system of proto-writing and proto-arithmetic in the sense defined above.

6.5 The Emergence of Proto-Cuneiform Bookkeeping as a Transformation Process

The extant proto-cuneiform tablets with their incipient form of writing and arithmetic provide us with a missing link between non-literate and literate societies. They show that the seemingly sudden emergence of writing and arithmetic at the turn of the fourth to the third millennium in Mesopotamia was actually the result of a complex transformation process. On the one hand, the proto-cuneiform system documents the end of a long-lasting historical process of transformation, encompassing several independent dimensions. On the other hand, it represents only the nucleus from which writing and arithmetic emerged.

The roots of both cultural techniques reach back into the Upper Palaeolitic when humans began to represent mental constructs by iconic or abstract symbols. What we know from ethnology about indigenous cultures indicates that, on this basis, the use of tallies in rural communities and probably even the use of limited counting sequences may have been established. However, for some 10,000 years, characterized by the globalization of agricultural, ceramic and metallurgical technologies, no remarkable further developments toward the invention of writing and arithmetic can be identified.

The change that can be observed in the second half of the fourth millennium can be conceived of as a transformation process that was triggered by the establishment of a redistributive economy in the context of the emergence of cities, and the stratification of the society in early state organizations. This transformation process started with an exploitation to their limits of the potentials of existing tools of symbolic representation, followed by a transfer of symbolically represented information to a new medium. Two types of independent information were concerned: the information represented by combinations of counters used to control quantities; and the information represented by seal impressions used to secure the objects of the administration. These types of information were transferred to this common medium by using sealed clay bullae with combinations of tokens inside and, finally, sealed clay tablets with numerical impressions.

The extant simple-shaped clay counters used in rural communities for controlling small quantities of resources and products were differentiated. The increased number of shapes corresponds to the new economical circumstances, which required greater quantities of more objects to be controlled. But the limitations created by such an exploitation of tools for a completely different social setting are obvious. Thus, it comes as no surprise that they soon underwent a transformation process.

The transformation of these tools started with a transfer of two types of independent information to a new common medium. The information represented by combinations of counters used to control quantities and the information represented by seal impressions used to secure the objects of the administration were transferred to this common medium by using clay bullae and, finally, clay tablets. The potential of these clay tablets, in particular, determined the further development toward proto-cuneiform writing. They enabled the represented amount of information to be extended and an increase in the number of semantic categories for different types of information. This new potential was first used by means of the numero-ideographic tablets to indicate the objects of economic transactions. To a greater degree than their preliterate precursors, such tablets became independent of the context in which they were written.

The potential was further realized by three innovations. First, the semantic categories of the newly introduced graphemes were extended so that they could cover all types of information needed for controlling the accumulation and distribution of resources and products. Second, the economic and administrative activities involved were modeled in terms of these categories, and representation structures were created for mapping the modeled activities onto the tablets. Third, the graphemes and formats created in this process of transforming information were standardized, as far as this was possible.

The administrative proto-cuneiform documents representing the majority of the texts of the proto-cuneiform corpus can thus be considered as the transformation of a mental model of the accumulation and distribution of resources and products into an external symbolic representation comprising formats for the major categories of economic information and rules for symbolic operations representing economic and administrative activities. This interpretation of the documents implies—as far as the administrative texts are concerned—that the development toward the proto-cuneiform writing system did not involve any substantial tendency to eliminate the dependency of either the semantic of the numerical notations, or of the additional graphemes on their function to control economical processes. Proto-cuneiform writing thus remained essentially a system of proto-writing, and the calculations performed with the context-dependent numerical notations remained essentially operations in proto-arithmetical systems. Both systems, of course, were incomparably more complex and powerful than their preliterate precursors.

This was not the end of the transformation process, however. The administrators of the early city states of Mesopotamia used proto-cuneiform tools exclusively to control economical transactions, but their potential to represent mental constructions reached far beyond this limited field of application. This implicit potential of proto-cuneiform to be further developed toward writing and arithmetic, however, was first noticeable only as a side effect of its main functions. The great number of graphemes with conventional meanings required some kind of institutional support for transferring the necessary knowledge for their use from one generation to the next. Such training institutions, then as now, do not realize economical goals, but rather teach how to use tools. Thus, we find attempts to generalize proto-cuneiform writing and its inherent techniques of operating with numerical notations specifically in a school context.

6.6 The Unexplored Transition from Proto-Writing and Proto-Arithmetic to Writing and Arithmetic

With regard to this further development, an atypical group of some 670 texts and fragments deserves closer attention. These represent standardized lexical lists (Englund and Nissen 1993) which are generally considered to be school texts. The lists contain entries ordered by semantic similarity. Some of them contain sign combinations related to resources and products such as animals, plants or manufactured products; others contain sign combinations related to geographical locations and persons. One list may even represent some kind of text of an oral rhetoric tradition. If these lexical lists did, in fact, serve to teach signs and sign combinations relevant for the later bookkeeping practice of the disciples, one would expect a high communality with the designations of resources and products that occurred frequently in the proto-cuneiform administrative documents. This, however, in general is not true. Simple sign combinations, which are recorded mainly at the beginning of a list, are often used in administrative texts as well. More complex sign combinations, however, can rarely be found in the corpus of administrative texts. The scribes who designed the lexical lists and probably used them for teaching purposes had a more sophisticated perspective in mind than to simply satisfy the immediate requirements of the administration's practitioners.

A similar tendency to depart from administrative purposes is characteristic of certain texts with calculations, which were obviously written in an educational context. Apart from their formats, which differ from the standard formats of administrative proto-cuneiform tablets, these texts characteristically contain problems that never occurred in practical contexts. One way to construct such problems was to use unrealistic numerical values, for instance, by asking the area of a field to be calculated, giving measurements that were much too large for any real field. Another way, which is attested only in later texts, however, was to reverse the problems of the practitioners. While the surveyors of the administration always measured the lengths and widths of fields and calculated the areas, in such a problem, the area of a field together with its length may be given and the task would be to calculate its width.

Such extensions of the main functions of administrative proto-cuneiform documents may have triggered the development of proto-cuneiform into cuneiform writing, and of proto-arithmetical techniques of calculation without numbers into the arithmetic of Babylonian mathematics. It is also possible, however, that specific achievements in the context of teaching and learning played only a minor role in the development toward writing and arithmetic. There were at least several other factors that may have initiated this development. The conditions which determined, in Mesopotamia in particular and in the wider area of neighboring regions in general, the further development of writing and arithmetic were, in fact, different from and much more complex than those that induced and constrained the creation of proto-cuneiform.

1. The proliferation of persons, institutions and locations to be identified made it increasingly necessary to find a coding principle (phonetization) according to which the symbolic coding and decoding of names could be simplified. (Charvát 2002)

2. The areas of application in which writing was used were extended from the documentation of economical activities, first to the support of memorizing orally transmitted texts such as hymns, incantations, proverbs and epic poems, later also to formalized texts with variable content, such as contracts and legal documents, and, finally, to freely composed texts such as letters.

3. The development in the third millennium BCE from the city states of the Early Dynastic Period (Ur, Shuruppak, Girsu, and so forth) to empires, which at times covered great parts of Mesopotamia (Sargonic empire, Ur III state), consequently had growing bureaucracies which brought about the specialization of scribal professions and the introduction of specialized terminologies and mathematical techniques used, in particular, in administrative units.

4. Proto-cuneiform was developed into cuneiform writing in a multilingual setting in which, in particular, Sumerian, a language with unknown provenance, and the semitic language Akkadian coexisted with alternating dominance.

5. Proto-cuneiform was not the only writing system to emerge at the end of the fourth millennium BCE. Two other writing systems were created which were related in different ways to proto-cuneiform: the proto-Elamite writing system of the highlands of Iran[20] and the system of Egyptian hieroglyphs together with its hieratic form.[21]

6. From the end of the third millennium and throughout the second millennium, cuneiform writing systems were created in the Levantine and Anatolian regions to the west and north west of Mesopotamia for several languages (Hurrian, Hittite, Hattic, Palaic, Luwian, Ugaritic). At the same time, to the east of Mesopotamia, cuneiform Elamite was created and used throughout the area until the second half of the first millennium when it was complemented by Old Persian cuneiform.

7. Around the same time, some completely different writing systems began to emerge and later disappeared again, partly in some of the regions to the north and to the east of Mesopotamia (Anatolian hieroglyphs, linear Elamite, Indus script), partly further west in the Mediterranean region, in particular on the islands of Cyprus and Crete (Cretan hieroglyphs, Linear A, Linear B, Cypro-Minoan syllabary). Furthermore, in the mid-second millennium in the Levantine region, the first alphabetic systems emerged (Proto-Sinaitic, Proto-Canaanite) followed by the Phoenician alphabet at the end of the millennium. Finally, the widespread alphabetic writing systems of the Arabic and the Greco-Roman world were created. These have survived until the present day.

[20]See Englund in (Houston 2004, 100–149; Dahl 2005).
[21]See Baines in (Houston 2004, 150–189; Wengrow 2006).

This brief survey of the conditions that induced and constrained the development of writing and arithmetic shows that the development of the various systems of writing and calculation cannot have followed a common pattern, but must have been different under different social, geographical and historical conditions. The development of writing and arithmetic depended on the interaction of different processes, such as:

1. the phonetization of a system of proto-writing, at the beginning allegedly (in the case of proto-cuneiform) by using the rebus principle, followed by the creation of standardized syllabaries or alphabets,
2. the generalization of the areas of application of a writing system, followed by a differentiation into segments, often with different lexicons, partly even with different syntax and different ways of constructing semantic relations as, for instance, in the case of the differentiation of writing and mathematics,
3. the adaptation of a writing system to a language other than the one it was created for,
4. the dissemination of writing and arithmetic by trade or by the migration of people,
5. the development of writing and arithmetic stimulated by the influence of one system on another,
6. the reinvention of techniques of writing or arithmetic triggered by the diffusion of incomplete information about a system that already existed,
7. the independent development of techniques of writing or arithmetic in different cultures with similar constellations with regard to the conditions that induced and constrained such development.

How writing and arithmetic developed in different geographical regions and under different historical conditions, and how they finally became globalized in the sense explained at the beginning of this paper, was determined by the interaction of such processes. Any explanation of specific historical developments of these cultural techniques has to take into account that they depended not only on the internal opportunities and constraints of the specific system of symbolic representation that was used, but also on interaction and exchange processes within and between cultures. From the viewpoint of this theoretical perspective, the development of writing and arithmetic from its beginnings to its globalization seems to be only insufficiently investigated. Disciplines such as archaeology, philology, linguistics and history of mathematics, which are concerned with aspects of this development, have contributed studies about the influence of specific conditions on specific developments, but the interdisciplinary integration of their results is still inadequate. Certain research deficits concern crucial details that require disciplinary research. These can only be identified by integrating the results of different disciplines.

As far as the origin of writing and arithmetic is concerned, in particular the origin of cuneiform writing, the situation can be briefly characterized in the following way. Three systems of writing (proto-cuneiform, proto-Elamite, Egyptian hieroglyphs) were created at nearly the same time, but their fates were different. Proto-cuneiform developed into cuneiform, which was disseminated and spread over great parts of the ancient Near East, influencing the development of other systems of writing, until it disappeared at some time in the first millennium CE. Proto-Elamite disappeared soon after its emergence; it was later replaced by cuneiform writing. Egyptian writing developed in parallel to the development of cuneiform writing into a full-fledged writing system for the Egyptian language. It survived along with cuneiform and disappeared at around the same time, but its use remained essentially restricted to Egypt itself. What was the reason for the near simultaneous emergence of the three systems? To what extent did they emerge independently of each other? How can their different fates be explained? How did the feedback of their different fates influence their internal development? These questions remain to a great extent unanswered, or the answers that are given are controversial. Some answers are commonly accepted, but are based on common-sense beliefs rather than a critical evaluation of the extant sources.

Concerning cuneiform writing in particular, it is well established that fully developed systems of writing and arithmetic existed at the latest in the first half of the second millennium BCE, in the Old Babylonian period. At this time, cuneiform writing was still used primarily for controlling economical activities, but in addition it was now applied to write down the tremendous corpus of Old Babylonian literature, letters and legal documents. Similarly, proto-arithmetical means and notations still played a major role in the context of economical administration, but Babylonian scribes had additionally created an unprecedented, powerful system of numerical notation: the sexagesimal positional system. This now developed into the esoteric system of Babylonian mathematics, independently of the development of writing. As a consequence, writing and arithmetic were no longer dependent on each other and their development was no longer constrained by their economical function.

Cuneiform writing and Babylonian arithmetic both show specific (from a modern point of view, odd) characteristics which they never completely lost. The logo-syllabic cuneiform writing system, as it was used for writing the Sumerian and the Akkadian languages, was and remained further based on a system that made extensive use of ideographs and graphemes representing syllables, most of which were phonetically polyvalent and, in addition, also homophonous to other graphemes. The resulting structural ambiguity could only be resolved by taking into account the syntactic and semantic context. The sexagesimal positional system of Babylonian mathematics was more seriously deficient since there was no sign for zero that could be used to indicate an empty position, and there was no way of indicating the absolute value of a numerical notation since there was no sign indicating the border between the whole number part and the fractional part of a notation. These characteristics of cuneiform writing and arithmetic require explanations to clarify how they emerged as an effect of the constrained development of the two cultural technologies.

Proto-cuneiform writing, on one hand, and Old Babylonian writing and arithmetic, on the other, mark the onset and the offset of complex developmental processes over a period of some 1,000 years. Many details of these processes have been successfully reconstructed, but they were mostly, if at all, interpreted in speculative historical narratives which are simplistic and often contradict knowledge achieved in other disciplines or by specialists working on another aspect of the historical process. What happened in the 1,000 years between the late Uruk and the Old Babylonian period still merits further study. Concerning the development of writing, for instance, it has been argued that the complex logo-syllabic structure of cuneiform writing resulted from the rebus principle, which allegedly determined the earliest stage of phonetization.[22] It is assumed that at a time when a stable syllabary did not yet exist, homophony between ideographic symbols for recognizable objects and phonemes occurring in names and abstract words, which could not be represented pictographically, was used ad-hoc to enable the symbolic representation of such names and objects. This would not only explain how ideographs were complemented with phonetic values, but also why the cuneiform writing system had so many homophonous and polyphonous graphemes. Rebus writing would automatically create homophony, since this is what it is based on, and polyphony, since the same object may have had different designations.

This simple explanation cannot be applied convincingly to the extant sources. The few examples of rebus writing that allegedly have been identified in proto-cuneiform texts are all problematic. But the next earliest group of texts, the texts of the ED IIIa period written around the middle of the third millennium BCE, already indicate the existence of a rudimentary, but stable, syllabary.[23] The assumption of a phase of rebus writing is thus merely a hypothetical construction concerning a time period from which no texts survive. But even if such a phase did exist, what could this explain? Why could the phonetic representation not remove the ambiguities allegedly inherited by the syllabary? Why was the syllabary never made less ambiguous at a later stage? Why should a hypothetic phase of rebus writing provide more important explanations than, for example, the transition from the representation of the Sumerian to that of the completely different Akkadian language, which had a proven influence on the construction of the syllabary of cuneiform writing?

The situation is similar in the case of the development of Babylonian arithmetic. There is a prejudice shared by philologists and historians of mathematics

[22] See Cooper in (Houston 2004, in particular 89–90).
[23] See Krebernik in (Bauer et al. 1998).

that the numerous forms of historically and geographically different numerical notations, which are characteristic of the cuneiform administrative documents, are only different symbolic representations of an underlying, common concept of number. This widespread prejudice materializes in transcription rules according to which the developing numerical notations of cuneiform sources are uniquely transcribed into modern Indo-Arabic numerals, or even into algebraic variables. Such transcriptions are of no use for any attempt to reconstruct the developing mental models underlying the development of numerical notations, which finally brought about the sophisticated—but deficient—arithmetic of Babylonian mathematics.

Looking at the broader picture of the globalization of writing and arithmetic, the situation is even more unsatisfactory. It is only too obvious that the spreading of writing and arithmetic in the ancient Near East and its neighboring regions resulted from various forms of cultural interaction and exchange.[24] As a consequence, the degree of mutual independence of the various systems and the ways in which they developed, under specific conditions, their specific structures and specific areas of application differed considerably between the early literate cultures. Only when the different ways in which systems of writing and arithmetic developed under existing constraints are reconstructed and explained can there be any hope of giving a convincing answer to the more general question of how often, and where, writing and arithmetic were created completely independently of each other.

References

Bauer, J., R. K. Englund, and M. Krebernik (1998). *Mesopotamien. Späturuk-Zeit und Frühdynastische Zeit.* Göttingen: Vandenhoeck & Ruprecht.

Charvát, P. (1993). *Ancient Mesopotamia. Humankind's Long Journey into Civilization.* Prague: Oriental Institute.

Charvát, P. (2002). *Mesopotamia Before History.* London: Routledge.

Culican, W. (1966). *The First Merchant Venturers: The Ancient Levant in History and Commerce.* Library of the Early Civilizations. London: Thames and Hudson.

Dahl, J. L. (2005). Animal Husbandry in Susa During the Proto-Elamite Period. *Studi Micenei ed Egeo-Anatolici 47*, 81–134.

Damerow, P. (1996). *Abstraction and Representation: Essays on the Cultural Revolution of Thinking*, Volume 175 of *Boston Studies in the Philosophy of Science.* Dordrecht: Kluwer.

[24]See (Culican 1966; Helck 1979; Gibson and Biggs 1991; Charvát 1993; Potts 1993; Sasson 1995, vol. 4, part 9; Potts 1997).

Damerow, P. (1998). Prehistory and Cognitive Development. In J. Langer and M. Killen (Eds.), *Piaget, Evolution, and Development*, pp. 247–269. Mahwah, NJ: Erlbaum.

Damerow, P. (2007). The Material Culture of Calculation. In U. Gellert and E. Jablonka (Eds.), *Mathematisation and Demathematisation*, pp. 19–56. Rotterdam: Sense Publishers.

Damerow, P. and R. K. Englund (forthcoming). *Materialien zu den frühen Schriftzeugnissen des Vorderen Orients*, Volume 3 of *The Proto-Cuneiform Texts from the Erlenmeyer Collection*. Berlin: Gebr. Mann Verlag.

DeFrancis, J. (1989). *Visible Speech: The Diverse Oneness of Writing Systems*. Honolulu: University of Hawai'i Press.

Dittmann, R. (1986). Seals, Sealings and Tablets: Thought on the Changing Pattern of Administrative Control from the Late-Uruk to the Proto-Elamite Period at Susa. In U. Finkbeiner and W. Röllig (Eds.), *Gamdat Nasr: Period or Regional Style*, pp. 332–336. Wiesbaden: Reichert.

Englund, R. K. (2006). An Examination of the "Textual" Witnesses to Late Uruk World Systems. In Y. Gong and Y. Chen (Eds.), *A Collection of Papers on Ancient Civilizations of Western Asia, Asia Minor and North Africa*, pp. 1–38. Beijing.

Englund, R. K. and H. J. Nissen (1993). *Die Lexikalischen Listen der Archaischen Texte aus Uruk. Bd.13. Archaische Texte aus Uruk*, Volume 3 of *Ausgrabungen der Deutschen Forschungsgemeinschaft in Uruk-Warka*. Berlin: Gebrüder Mann.

Gelb, I. J. (1952). *A Study of Writing: The Foundations of Grammatology*. Chicago: University of Chicago Press.

Gibson, M. and R. D. Biggs (Eds.) (1991). *The Organization of Power: Aspects of Bureaucracy in the Ancient Near East* (2. ed.). Chicago: The Oriental Institute of the University of Chicago.

Helck, W. (1979). *Die Beziehungen Ägyptens und Vorderasiens zur Ägäis bis ins 7. Jahrhundert v. Chr*, Volume 120 of *Erträge der Forschung*. Darmstadt: Wissenschaftliche Buchgesellschaft.

Houston, S. D. (Ed.) (2004). *The First Writing: Script Invention as History and Process*. Cambridge: Cambridge University Press.

Hyman, M. D. (2006). Of Glyphs and Glottography. *Language and Communication 26*(3–4), 231–249.

Leroi-Gourhan, A. (1993). *Gesture and Speech*. Cambridge Mass.: MIT Press.

Michalowski, P. (1994). Writing and Literacy in Early States: A Mesopotamianist Perspective. In D. Keller-Cohen (Ed.), *Literacy: Interdisciplinary Conversations*, pp. 49–70. Cresscill: Hampton Press.

Nissen, H. J., P. Damerow, and R. K. Englund (1993). *Archaic Bookkeeping: Early Writing and Techniques of Economic Administration in the Ancient Near East.* Chicago: Chicago University Press.

Potts, D. T. (1997). *Mesopotamian Civilization: The Material Foundations.* Ithaca: Cornell University Press.

Potts, T. F. (1993). Patterns of Trade in Third-Millennium BC Mesopotamia and Iran. *World Archaeology 24*(3), 379–402.

Renn, J. and P. Damerow (2007). Mentale Modelle als kognitive Instrumente der Transformation von technischem Wissen. In H. Böhme, C. Rapp, and W. Rösler (Eds.), *Übersetzungen und Transformationen*, Volume 1 of *Transformationen der Antike*, pp. 311–331. Berlin: De Gruyter.

Robson, E. (2008). *Mathematics in Ancient Iraq: A Social History.* Princeton, NJ: Princeton University Press.

Sasson, J. M. (Ed.) (1995). *Civilizations of the Ancient Near East.* New York: Charles Scribner's Sons.

Schmandt-Besserat, D. (1992). *Before Writing.* Austin: University of Texas Press.

Wengrow, D. (2006). *The Archaeology of Early Egypt: Social Transformations in North-East Africa, 10,000 to 2650 BC.* Cambridge World Archaeology. Cambridge: Cambridge University Press.

Wertheimer, M. (1925). Über das Denken der Naturvölker: Zahlen und Zahlgebilde. In M. Wertheimer (Ed.), *Drei Abhandlungen zur Gestalttheorie*, pp. 106–163. Erlangen: Verlag der philosophischen Akademie.